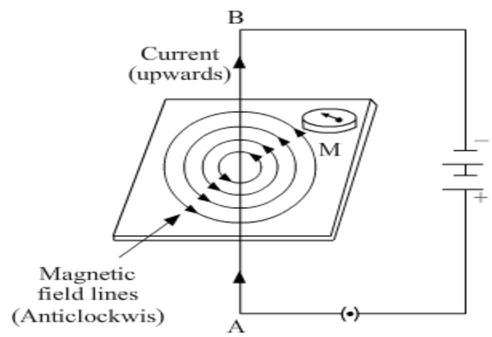
Magnetic forces in daily life

- A door catch is a simple device that uses the magnetic force of attraction to hold a door closed – Eg. Refrigerator
- Use of magnetic force - Televisions, radios, microwave ovens, telephone systems, Speakers and computers. Computer hard drives use magnetism to store the data on a rotating disk
- An industrial application of magnetic force is an electromagnetic crane that is used for lifting metal objects.
- Electric motors use the electromagnetic force between a magnet and a current carrying coil to produce movement.
- Electric generators use the electromagnetic force between a magnet and a moving coil to generate electrical energy.

Magnetic Field

Introduction : H. C. Oersted in the year 1820 discovered the magnetic effect of electric current. According to this effect, when a current flows through a conductor, a magnetic field is produced in the region surrounding the conductor. He observed that a tiny magnetic needle placed near a straight conductor deflected when a current flows through it.



An electric charge in motion produces magnetic field along with electric field. A stationary charge produces only electric field.

Concept of Magnetic lines of force : The magnetic field generated by the current in a conductor is visualized with the concept of **magnetic lines of force or magnetic flux**. They are imaginary curves with the properties as,

(1) they are continuous closed loops,

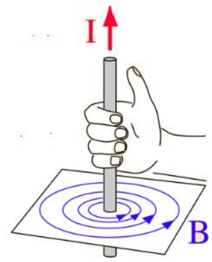
(2) The direction of magnetic field at a point is given by the tangent drawn to the line of force at that point.

(3) The number of magnetic lines of force normal to a given surface is the magnetic flux and its unit is weber (Wb).

(4) The magnetic flux per unit area is the strength of magnetic field at a point. Its unit is Wbm^{-2}. It is also called as **Magnetic flux density.**

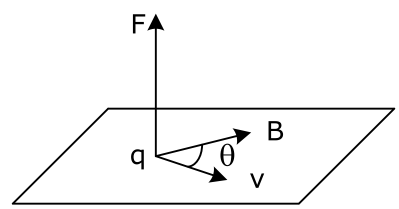
(5) The magnetic field lines due to a straight current carrying conductor is concentric circles in a plane perpendicular to the conductor as shown.

The direction of the magnetic field at a point due to current in a conductor is given by the following rules



1. **Ampere's swimming rule :** If a swimmer is imagined to swim along the conductor in the direction of current, facing the needle, then the north pole of the needle is deflected to his left.
2. **Right hand clasp rule :** The conductor is imagined to be held in the right hand so that the thumb points in the direction of current, then the direction in which the other fingers curl around the conductor gives the direction of the magnetic field.
3. **Maxwell's cork screw rule :** If a right handed cork screw is rotated so that it moves in the direction of flow of current through the conductor then the direction of rotation of the head of the screw gives the direction of magnetic field.

Force on a charged particle moving in a magnetic field



Consider a positively charged particle q moving with a velocity v in a magnetic field of strength B as shown in the diagram. The force experienced by the charge is given by $\vec{F} = q\left(\vec{v} \times \vec{B}\right)$ or $\vec{F} = qvB\sin\theta\,\hat{n}$.

Thus

(1) The magnitude of force $\vec{F}$ is directly proportional to the magnitude of the magnetic field applied,

(2) The magnitude of force is directly proportional to the magnitude of the charge

(3) The magnitude of force is directly proportional to the velocity of the charge.

(4) The direction of this force is perpendicular to both v and B in accordance with the cross product of vectors.

Special Cases :

Consider F = qvBsin θ. If v = 0, F = 0 i.e., a charged particle at rest in a magnetic field experiences no force.

(1) If θ = 0° or 180°, F = 0 i.e., a charged particle moving parallel or antiparallel to the direction of the field experiences no force.

(2) If θ = 90°, F will be maximum and it's value is given by $\mathbf{F_{max} = qvB}$.

Fleming's left hand rule The direction of the charged particle moving at right angles to the magnetic field is given by Fleming's left hand rule. According to this rule

"If the first three fingers of the left hand i.e., the thumb, the forefinger and the middle finger are stretched such that they are mutually perpendicular to one another and that the fore finger is in the direction of the field, the middle finger in the direction of velocity of positively charged particle, then the thumb gives the direction of the force on the charged particle".

Definition of magnetic field (B)

In the equation $F = qvB\sin\theta$, $B = F$, if $q = 1$, $v = 1$ and $\theta = 90°$. Then "the **magnetic field** at a point is numerically equal to the force on a unit positive charge moving through the point in a direction perpendicular to the field with unit velocity".
The SI unit of magnetic field B is **tesla (T)** or weber/metre^2 (Wbm^{-2})

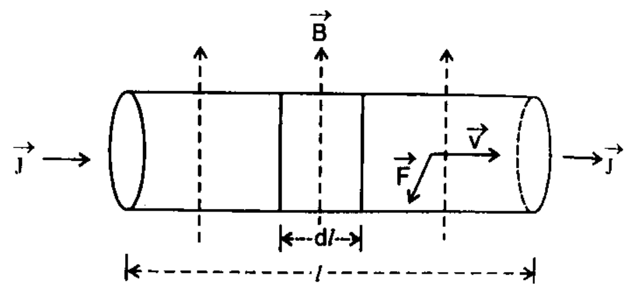
In the equation $F = qvB \sin \theta$. $B = 1$ T if $F = 1$ N, $q = 1$ C, $v = 1\ ms^{-1}$ and $\theta = 90°$. The magnetic field at a point is said to be 1 tesla if 1C of positive charge moving through the point at right angles to the field with a velocity of $1ms^{-1}$ experiences a force of 1N.

Note :

- An electric charge q moving with a velocity v in a region having magnetic field B and electric field E will experience a resultant force $\vec{F}=q\vec{E}+q\left(\vec{v}\times\vec{B}\right)=q\left(\vec{E}+\vec{v}\times\vec{B}\right)$. This relation is called **Lorentz relation** and the resultant force is called the **Lorentz force.**
- A charge moving at right angles to a uniform magnetic field will describe a circular path obeying the relation $\frac{mv^2}{r}=qvB$. If the angle between the direction of velocity of charge and B is less than 90^0 then the path of the charge is a helix.
- As the force acts along a direction at right angles to direction of motion of the charge, the work done is zero. Thus there is no change in magnitude of velocity of the charge but only the direction changes.

Force on a currrent carrying conductor in a magnetic field

Consider a conductor carrying current I. Let the magnetic field B be acting on the conductor which is in the plane of the paper and upwards.
If N be the number of charges per unit volume, then the number of charges in the volume dV is given by $\boldsymbol{NdV}$
The force due to these charges is $\boldsymbol{dF = NdV(qv \times B)}$ **....(1)**



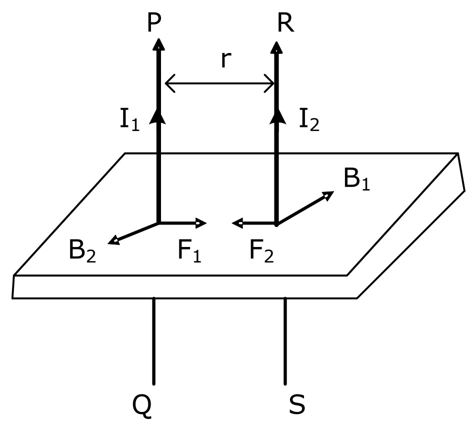
where $(qv \times B)$ is the force acting on a charge moving with velocity v.
Since the current I is steady, all the charges are assumed to be moving with the same velocity v.
But $\boldsymbol{Nqv = J}$ called the current density which is the current per unit area.
(*Since*** $N = \frac{n}{dV} = \frac{n}{A\,dl}$ **,** $As\ I = \frac{nq}{dt}$, $q = \frac{I\,dt}{n}$ $and\ v = \frac{dl}{dt}$ $J = \frac{I}{A}$ **)**

Thus equation (1) is $dF = (J \times B)\,dV$ ....(2)
As $dV = A\,dl$ where dl is a small elemental length of the conductor.
Equation (2) is $dF = (J \times B)\,A\,dl$ ....(3)
But current density is $J = \frac{I}{A}$
Thus equation (3) is $dF = (I\,dl \times B)$ ......(4)
By integrating equation (4), the total force on the conductor of length l is
$F = \int dF = \int (I\,dl \times B)$ or $\boldsymbol{F = I(l \times B)}$ ....(5)
The magnitude of the force is given by $\boldsymbol{F = BIL\sin\theta}$ where θ is the angle between the direction of current and magnetic field.
Thus the magnitude of the force depends on the (1) magnitude of the current it carries, (2) the strength of the magnetic field and (3) the length of the conductor.
Also the force is maximum when the current flows perpendicular to the direction of magnetic field ($F_{max} = BIL$) and it is zero when current flows parallel to magnetic field.

Consider a magnetic field B acting along the plane of the paper upwards as shown. If a current of I ampere flows through the conductor, then the direction of force on the conductor is outwards and perpendicular to plane of the paper as shown.
The direction of this force is given by **Fleming's left hand rule**.
"If the thumb, the forefinger and the middle finger are stretched mutually perpendicular to one another and if the fore finger is in the direction of magnetic field, the middle finger in the direction of current, then the thumb gives the direction of the force on the conductor".

Expression for force between two infinitely long parallel conductors carrying currents

Consider two long straight conductors PQ and RS carrying currents I_1 and I_2, placed parallel and close to each other separated by a distance r. Each conductor is in the magnetic field of the other and hence they experience mechanical force. The magnetic field at any point on the conductor RS due to the current I_1, in PQ is $B_1 = \frac{\mu_0 I_1}{2\pi r}$ ......(1),



The direction of the magnetic field acts in a direction perpendicular to the plane of the diagram and inwards according to right hand clasp rule. The conductor RS experiences a mechanical force because of this magnetic field and this force is given by

$F_2 = B_1 I_2\, l \sin\theta$.

$F = B_1 I_2 l$ ....(2) ($\because \theta = 90°$) where l is the length of either of the conductors.

From (1) and (2), $F_2 = \frac{\mu_0 I_1 I_2\, l}{2\pi r}$ .....(3)

The direction of this force is given by Fleming's left hand rule and acts in the plane of the diagram towards PQ.

Similarly the magnetic field at any point on the conductor PQ due to the current I_2, in RS is $B_2 = \frac{\mu_0\, I_2}{2\pi r}$ ......(4),

The force on the conductor PQ, exerted by the magnetic field B_2 due to the current I_2 in RS is given by $F_1 = B_2 I_1\, l$ ....(5) . Thus from (4) and (5) the force is

$F_1 = \frac{\mu_0 I_1 I_2\, l}{2\pi r}$ ....(6)

The direction of this force acts in the plane of the diagram towards RS.

From equations (3) and (6) it is observed that $F_1 = F_2$..

If $F = F_1/l$ called the force per unit length, then $F = \frac{\mu_0 I_1 I_2}{2\pi r}$ The conductors attract each other because of these mutual forces.

If the conductors carry currents in the opposite directions then the conductors will experience forces away from each other. Thus the conductors repel each other.

Definition of ampere Consider $F = \frac{\mu_0 I_1 I_2}{2\pi r} Nm^{-1}$. If $I_1 = I_2 = 1A$ and r = 1m, and conductors being placed in vacuum, $F = \frac{4\pi \times 10^{-7}}{2\pi} Nm^{-1}$. Ampere is defined as that steady current which when flowing through each of two infinitely long straight parallel conductors of negligible cross section separated by a distance of 1m in vacuum causes a force of 2 x 10^{-7} N per metre length of each conductor.

Biot-Savart law or Laplace's Law

Statement : The magnetic field at a point due to a current element at distance *r* from it is

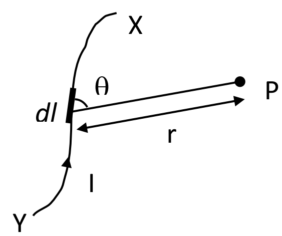
(1) directly proportional to the strength of current passing through the element of the conductor (I)

(2) directly proportional to the length of the current element (dl)

(3) directly proportional to the sine of the angle between the element and the line joining the element and the point (sin θ)

(4) inversely proportional to the square of the distance between the current element and the point (r^2)

Explanation



Consider a conductor XY carrying current I. Let dl be a small current element. Let P be a point at a distance r from the element.

According to Biot Savart's law, the magnetic field dB at P due to dl is

given by $dB \propto \frac{I\, dl\, sin\,\theta}{r^2}$ **or** $dB = K \frac{I\, dl\, sin\,\theta}{r^2}$

where K is a constant. In SI units K = $\frac{\mu_0}{4\pi}$ where μ_0 is the absolute permeability of free space. Here θ is the angle between $\vec{dl}$ and $\vec{r}$ and $\mu_0 = 4\pi \times 10^{-7}\, Tm\, A^{-1} \left(H\, m^{-1}\right)$.

Thus $dB = \frac{\mu_0}{4\pi} \cdot \frac{I\, dl\, sin\,\theta}{r^2}$ Wbm^{-2}.

In vector notation, $\vec{dB} = \frac{\mu_0 I}{4\pi} \cdot \frac{\vec{dl} \times \vec{r}}{r^3} = \frac{\mu_0 I}{4\pi} \cdot \frac{\vec{dl} \times \hat{r}}{r^2}$.

Since $d\vec{B}$ is the cross product of $d\vec{l}$ and $\vec{r}$, the direction of $d\vec{B}$ is along the normal (or perpendicular) to the plane containing $d\vec{l}$ and $\vec{r}$. This is also called Laplace law.

To arrive at Biot Savart's law

Consider a conductor XY of area of cross section A carrying current I. Consider a small element of length dl. Then the volume of the element, $dV = A\,dl$. Let P be a point at a distance r from the element.

Let the charges flow in the conductor with a velocity v. Let n be the number of charges per unit volume. Let each charge be given by q.

Then the total charge flowing through the element is $dQ = nq\,dV$

$\Rightarrow \quad dQ = nqA\,dl$ ----------- (1)

Magnetic field at P due to the element is $\overrightarrow{dB} = \frac{\mu_0}{4\pi}\frac{dQ\,(\vec{v}\times\vec{r})}{r^3}$

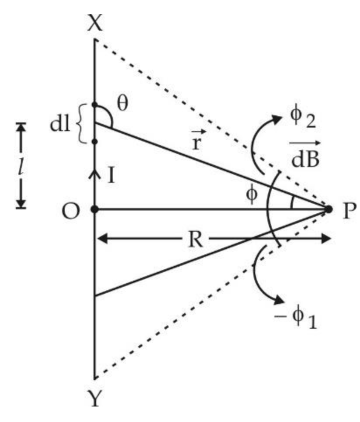
$\overrightarrow{dB} = \frac{\mu_0}{4\pi}\frac{nqAdl\,(\vec{v}\times\vec{r})}{r^3}$ or $\overrightarrow{dB} = \frac{\mu_0}{4\pi}\frac{nqA\vec{v}(\overrightarrow{dl}\times\vec{r})}{r^3}$

But $I = \frac{dQ}{dt} = nqA\frac{dl}{dt}$ or $I = n\,q\,Av \therefore \overrightarrow{dB} = \frac{\mu_0}{4\pi}\frac{I(\overrightarrow{dl}\times\vec{r})}{r^3}$

Magnitude of dB is $dB = \frac{\mu_0}{4\pi}\frac{I\,dl\,r\sin\theta}{r^3}$ or $\boxed{dB = \frac{\mu_0}{4\pi}\frac{I\,dl\sin\theta}{r^2}}$ dB is perpendicular to the plane containing $\overrightarrow{dl}$ and $\vec{r}$.

Expression for Magnetic field at a point due to a straight current carrying conductor of finite length

XY is a straight conductor carrying current I. To find the magnetic field at a point P distant R from the conductor, consider an element of length dl of the conductor at a distance l from O as shown in the figure.



According to Biot savart law, the magnetic field at P due to dl is given by $\Rightarrow \quad dB = \frac{\mu_0}{4\pi}\frac{I\,dl\sin\theta}{r^2}$

$\overrightarrow{dB}$ is perpendicular to the plane containing $\overrightarrow{dl}$ and $\vec{r}$ (plane of paper) away from the observer. The resultant magnetic field at P due to the entire conductor is

$B = \int dB = \int \frac{\mu_0}{4\pi}\frac{I\,dl\,sin\theta}{r^2} = \frac{\mu_0 I}{4\pi}\int\frac{dl\,sin\theta}{r^2}$ ....(1)

From the figure, $r^2 = l^2 + R^2 \quad \Rightarrow \quad r = \left[l^2 + R^2\right]^{1/2}$

Also $\sin\theta = \sin(180-\theta) = \frac{R}{r}$ and $\tan\phi = \frac{l}{R} \Rightarrow l = R\tan\phi$

Differentiating the above equation $\therefore dl = R\sec^2\phi\, d\phi$

Substituting for *dl, sinθ and r* from above equations in (1), we get

$$\therefore\ B=\frac{\mu_o I}{4\pi}\int_{-\phi_1}^{+\phi_2}\frac{R\sec^2\phi\, d\phi\, R}{r^3}=\frac{\mu_o I}{4\pi}\int_{-\phi_1}^{-\phi_1}\frac{R^2\sec^2\phi\, d\phi}{\left[l^2+R^2\right]^{3/2}}=\frac{\mu_o I}{4\pi}\int_{-\phi_1}^{+\phi_2}\frac{R^2\sec^2\phi\, d\phi}{\left[R^2\left(\tan^2\phi+1\right)\right]^{3/2}}$$

$$=\frac{\mu_o I}{4\pi R}\int_{-\phi_1}^{-\phi_2}\frac{\sec^2\phi\, d\phi}{\left(\sec^2\phi\right)^{3/2}}=\frac{\mu_o I}{4\pi R}\int_{-\phi_1}^{\phi_2}\cos\phi\, d\phi=\frac{\mu_o I}{4\pi R}\left[\sin\phi_2-\sin(-\phi_1)\right]=\frac{\mu_o I}{4\pi R}\left[\sin\phi_2+\sin\phi_1\right]$$

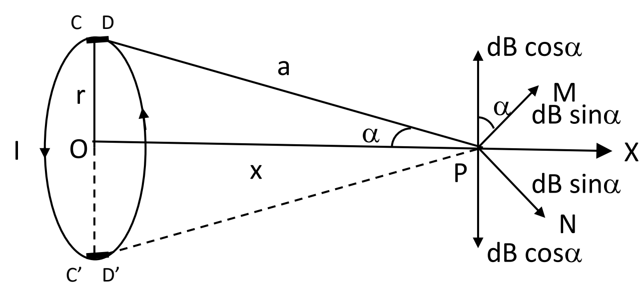
$\therefore\ \boxed{B=\frac{\mu_o I}{4\pi R}\left[\sin\phi_2+\sin\phi_1\right]}$ This is the expression for magnetic field at a point due to a straight conductor of finite length.

Special case : Conductor of infinite length

Assuing the length of the wire to be very large compared to the distance R from the point, we have $\phi_2=\frac{\pi}{2}$ and $\phi_1=\frac{\pi}{2}$. Substituting this condition in the above equation, $\therefore\ B=\frac{\mu_o I}{4\pi R}[2]\ \Rightarrow$ $\boxed{B=\frac{\mu_o I}{2\pi R}}$

Expression for magnetic field at a point on the axis of a circular coil carrying current :

Consider a circular coil of radius r carrying a current I as shown in the diagram. The plane of the coil is perpendicular to the plane of the diagram so that its axis lies in the plane of the diagram. Let P be a point on its axis OX at a distant x from the centre O of the coil. Consider a pair of diametrically opposite small elements CD and $C'D'$ each of length *dl*. The field at P due to CD is (from Laplace's law)



$dB=\frac{\mu_0}{4\pi}\frac{I\,dl\sin\theta}{a^2}$ (θ is the angle made by the element CD with the line joining centre of CD and the point P) or $dB=\frac{\mu_0}{4\pi}\frac{I\,dl}{a^2}$ (since θ = 90⁰ and sin90 = 1). The direction of magnetic field dB at P due to CD is along PM which is at right angles to the plane containing CD and point P. Similarly the direction of dB at P due to C'D' is along PN.

The field dB along PM is resolved into dBsinα along X direction and dBcosα along Y direction. The field dB along PN is resolved into dBsinα along X direction and dBcosα along - Y direction. As the components along the Y direction are equal and opposite, they cancel each other.

Thus net field at P due to elements CD and C'D' is 2dBsinα.

The resultant magnetic field at P due to the entire coil is given by $B = \sum 2\, dB \sin\alpha$

.

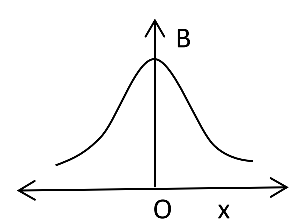
From the diagram $\sin\alpha = \frac{r}{a}$ and substituting for dB we get

$$B = \sum 2\left(\frac{\mu_0}{4\pi}\frac{I\,dl}{a^2}\right)\frac{r}{a} = \sum 2\left(\frac{\mu_0}{4\pi}\frac{I\,dl\,r}{a^3}\right) = \frac{\mu_0}{4\pi}\frac{2Ir}{a^3}\sum dl$$

$\sum dl = \pi r$ (half the circumference as the field is due to two diametrically opposite elements of the circular loop)

Thus $B = \frac{\mu_0}{4\pi}\frac{2\pi I r^2}{a^3}$. From the diagram $a^2 = r^2 + x^2 \; or \; a^3 = \left(r^2 + x^2\right)^{\frac{3}{2}}$ and if the coil contains n number of turns then $B = \frac{\mu_0}{4\pi}\frac{2\pi n I r^2}{\left(r^2 + x^2\right)^{\frac{3}{2}}}$.

The direction of B is along the axis of the coil and directed towards the observer facing the coil.



<u>Special case</u> Magnetic field at the centre of the coil carrying current.

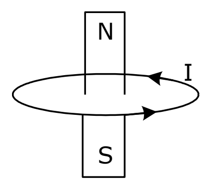
At the centre of the coil, x = 0. Thus from the above expression, the field at the centre of the coil is given by $B = \frac{\mu_0}{4\pi}\frac{2\pi n I}{r}$. The magnetic field at the centre of the coil is maximum and decreases on either side along its axis as shown the graph.

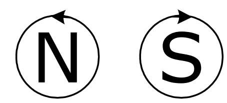
A current carrying coil as a magnetic dipole The magnetic field at a point on the axis of the coil is given by $B = \frac{\mu_0}{4\pi}\frac{2\pi n I r^2}{\left(r^2 + x^2\right)^{\frac{3}{2}}}$.

If the point is at a large distance compared to the radius of the coil, then x >> r, then r can be neglected in the above equation which reduces to the form $B = \frac{\mu_0}{4\pi}\frac{2\pi n I r^2}{x^3}$.

Consider a single coil or a loop (n = 1) carrying current of *I* ampere. The area of the coil is A = πr^2 and the product IA = M is called the **magnetic dipole moment** of the current loop.

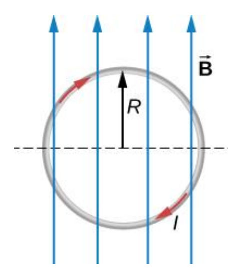
Thus the above equation becomes $B = \frac{\mu_0}{4\pi}\frac{2IA}{x^3}$. *Thus* $B = \frac{\mu_0}{4\pi}\frac{2M}{x^3}$. This equation can be compared to the equation of electric field at a point due to a electric dipole which is given by $E = \frac{\mu_0}{4\pi}\frac{2p}{x^3}$. Thus the electric field due to an electric dipole and magnetic field due to current loop at a point are inversely proportional to the cube of the distance.





Thus M is the magnetic analog of electric dipole moment p. M is called the magnetic dipole moment and its direction is at right angles to the plane of the coil. Hence a current carrying loop acts as a magnetic dipole. The current loop can be imagined to be a tiny magnet with one face behaving as a north pole and the other face behaving like a south pole as shown.

Force on a circular current loop in a magnetic field



A circular current loop of radius *R* carrying a current *I* is placed in the *xy*-plane. A constant uniform magnetic field cuts through the loop parallel to the *y*-axis (diagram). The loop is perpendicular to the direction of the magnetic field.

Consider the upper half of the loop. Consider a small element dl, then the magnetic force on it is given by $dF = IBdl\ sin\theta$ where θ is the angle between the element dl and the direction of the magnetic field.
Also $dl = R\ d\theta$ Thus $dF = IBR\ sin\theta\ d\theta$
The magnetic force due to the entire upper half of the loop is given by integrating the above equation

$$F_1 = \int dF = \int_0^{\pi} IBR\ sin\theta\ d\theta = IBR(-cos\pi + cos0) = 2IBR$$

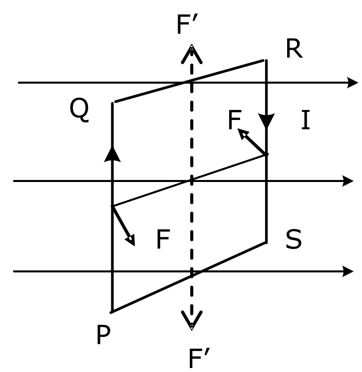
Similarly the magnetic force on the lower half loop is

$$F_2 = \int dF = \int_{\pi}^{0} IBR\, sin\theta\, d\theta = IBR(-cos0 + cos\pi) = -2IBR$$

The net force is the sum of these forces, $F = F_1 + F_2 = 2IBR - 2IBR = 0$.
Thus the net magnetic force acting on the loop is zero.

Torque on a current loop placed in an uniform magnetic field

Consider a coil PQRS carrying a current I placed in a uniform magnetic field B with it's plane at an angle θ with B as shown in the diagram. Let l be the length of the coil (PQ or RS), b the breadth of the coil and A the area of the coil. The forces experienced by QR and PS are $F' = B\,I\,b\,sin\theta$ each. These forces are equal in magnitude, opposite in direction, and act along the same line. Thus their resultant is zero.



The force on PQ is $F = I\left(\vec{l} \text{ x } \vec{B}\right) = BIl\sin 90° = BIl$

Similarly the force on RS is also $F = B\,I\,l$. These two forces are equal, parallel and do not act along the same line. Therefore they constitute a couple. The moment of this couple or torque on the current loop is τ = one of the forces x arm of the couple

Arm of the couple is b cosθ. ∴τ = $B\,I\,l$ cos θ. As l x b = A, the area of the loop, ∴τ = (I A) B cos θ.

But $IA = M$, the magnetic dipole moment of the loop. ∴τ = M B cos θ

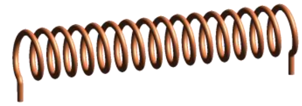
If α be the angle between the normal to the plane of the coil and the direction the field, then α = 90 - θ or θ = 90 - α.

∴τ = M B sinα or $\tau = \vec{M} \text{ x } \vec{B}$

If θ = 0° or α = 90°, the plane of the coil is parallel to B. Then the torque is maximum. τ_{max} = MB = $I\,A\,B$.

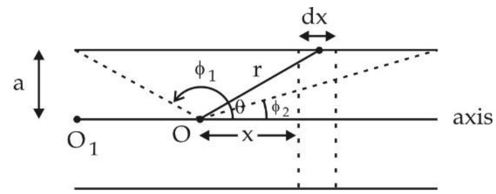
If θ = 90° of α = 0°, the plane of the coil is perpendicular to field. Then τ = 0

Solenoid A solenoid is a coil wound into a tightly packed helix. The term was invented by French physicist André-Marie Ampère to designate a helical coil.



Expression for Magnetic field at a point along the axis of a solenoid of finite length carrying current

The following figure shows the sectional view of the solenoid carrying current I. Let 'a' be its radius and let the number of turns per unit length of the solenoid be n. Consider a small segment of length dx of the solenoid. Then number of turns in this segment (element) $= n\,dx$.



Magnetic field at O due to this element is given by

$dB = \frac{\mu_o}{2}\frac{I\,n\,dx\,a^2}{\left(x^2+a^2\right)^{3/2}}$ This field is along the axis of the solenoid.

From the figure, $\tan\theta = \frac{a}{x}$ and $x^2 + a^2 = r^2$, $\sin\theta = \frac{a}{r}$

$$\Rightarrow \quad x = \frac{a}{\tan\theta} = a\cot\theta \qquad \therefore dx = -\,a\ \mathrm{cosec}^2\theta\ d\theta$$

$$\therefore dB = -\frac{\mu_o I}{2}\frac{n\,a^3\mathrm{cosec}^2\,\theta\ d\theta}{r^3} = -\frac{\mu_o n I}{2}\frac{a^3}{r^3}\mathrm{cosec}^2\theta\ d\theta \Rightarrow dB = -\frac{\mu_o n I}{2}\sin^3\theta\ \mathrm{cosec}^2\,\theta\ d\theta$$

$$= -\frac{\mu_o n I}{2}\sin\theta\ d\theta$$

Total magnetic field at O due to the entire solenoid is

$$B = \int dB = -\frac{\mu_o n I}{2}\int_{\phi_1}^{\phi_1}\sin\theta\ d\theta$$

$$B = -\frac{\mu_0\,n\,I}{2}\left[-\cos\theta\right]_{\varphi_1}^{\varphi_2} \text{ or } B = -\frac{\mu_0\,n\,I}{2}\left[-\cos\varphi_2 + \cos\varphi_1\right]$$

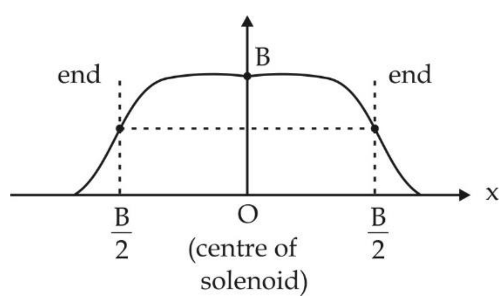
$$\boldsymbol{B = \frac{\mu_0\,n\,I}{2}\left[\cos\varphi_2 - \cos\varphi_1\right]}$$

<u>Special cases</u>

(i) If the solenoid is very long and O is in the mid point then $\varphi_1 = \pi$ and $\varphi_2 = 0$.

$$\therefore\ B = \frac{\mu_o n I}{2}(1+1) \Rightarrow \boxed{B = \mu_o\,n I}$$

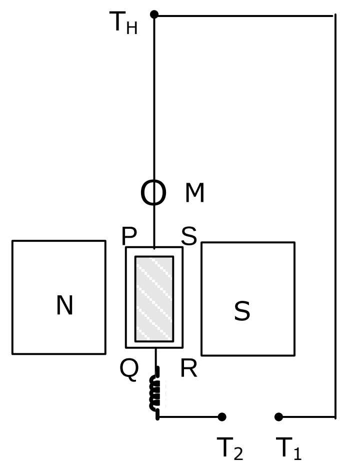
(ii) If O is at one end of the solenoid (say at O_1), then $\varphi_1 = \frac{\pi}{2}$ and $\varphi_2 = 0$.



$$\therefore B = \frac{\mu_o n I}{2}(0+1) \Rightarrow \boxed{B = \frac{\mu_o n I}{2}}$$

The graph indicates that the magnetic field is uniform along the axis inside the solenoid. At the end field $= \frac{B}{2}$.

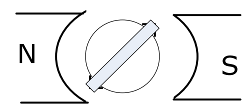
Moving coil Ballistic galvanometer :



It is a device used to measure the charge flowing through it as a sudden discharge. It also measures small currents of the order of micro ampere.

It is based on the principle that a current carrying loop placed in a uniform magnetic field experiences mechanical force and hence torque.

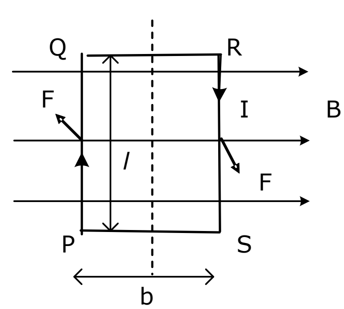
Construction : PQRS is a rectangular coil having n number of turns of copper wire wound on a nonmagnetic frame, suspended between the two concave pole pieces N and S of a powerful cylindrical permanent horse-shoe magnet, by means of a thin and long wire of phosphor bronze.



The upper end of the suspension wire is fixed to a torsion head T_H which in turn is connected to the terminal T_1, The lower end of the coil is connected to the terminal T_2 through a hair spring. A soft cylindrical iron is placed inside the coil to increase the strength of the magnetic field and to give radial field. A mirror M is attached to the suspension wire close to the coil.

The whole arrangement is enclosed in a non-metallic case and the base of the case is provided with leveling screws. With the help of a lamp and scale arrangement and the mirror the deflection of the galvanometer can be read.

Theory :



Consider the coil PQRS as shown in the diagram. Let n be the number of turns, l the length, b the breadth and A the area of the coil.

Let a charge Q be passed through the coil, then the instantaneous value of current is given by $\frac{dQ}{dt}$.

Initially the plane of the coil is parallel to the magnetic field B. The portion QR and PS of the coil do not experience any force.
The force on PQ is $F = n\,B\,i\,l$.
From Fleming's left hand rule, the direction of the force on PQ is perpendicular to the plane of the diagram and into it. Similarly an equal force acts on RS but in the opposite direction which is normal to plane of the paper and is towards the observer. These two forces constitute a couple due to which the coil deflects.
The moment of the couple or the torque is given by
$\tau = F \times b = n\,B\,i\,l \times b = n\,B\,i\,A$
If the current i acts for a short interval dt, then the moment of impulse is
Impulse of force = $n\,B\,i\,A\,dt$
Total moment of impulse = $\int_0^T n\,B\,i\,A\,dt$
This is moment of momentum= $\int_0^T n\,B\,A\,idt = nAB\int_0^T idt = nABQ$ ....(1) where $\int_0^T idt = Q$
But the moment of momentum = angular momentum = $I\,\omega$....(2)
Where I is the moment of inertia and ω is the angular velocity acquired by the coil due to rotation of the coil.
Thus $nABQ = I\,\omega$ .....(3)
Due to angular velocity, the coil has kinetic energy $\frac{1}{2}I\omega^2$ and the coil is brought to rest by performing work in twisting the wire by angle .
If the couple per unit twist is c, then couple for a twist = $c\theta$.
Work done for additional twist $d\theta$ is equal to $c\theta\, d\theta$.
Total work done is $\int_0^\theta c\theta\, d\theta = \frac{1}{2}\,c\theta^2$.
This work done is equal to the kinetic energy of oscillating system. Thus
$\frac{1}{2}\,c\theta^2 = \frac{1}{2}I\omega^2$ or $I\omega^2 = c\theta^2$ ......(4)
If T is the time period of oscillation of the coil,
$T = 2\pi\sqrt{\frac{I}{c}}$ or $T^2 = \frac{4\pi^2 I}{c}$ or $I = \frac{T^2 c}{4\pi^2}$ .... (5)
Multiplying equations (4) and (5) , $I^2\omega^2 = \frac{c^2\theta^2T^2}{4\pi^2}$ or $I\omega = \frac{c\,\theta\,T}{2\pi}$ .....(6)
Comparing (3) and (6), $nABQ = \frac{c\,\theta\,T}{2\pi}$
Thus $\boldsymbol{Q = \frac{T}{2\pi}\frac{c}{nAB}\,\theta}$ As $\frac{T}{2\pi}\frac{c}{nAB} = K$ is the Ballistic reduction factor of the galvanometer. Thus $\boldsymbol{Q = K\,\theta}$.

Charge sensitivity is the deflection produced per unit charge on the scale at a distance of 1 m from the galvanometer.

It is given by $\frac{\theta}{Q} = \frac{2\pi}{T}\frac{nAB}{c}$.

Current sensitivity is the deflection produced per unit current the scale at a distance of 1 m from the galvanometer. It is given by $\frac{\theta}{I} = \frac{nAB}{c}$

Thus charge sensitivity= $\frac{2\pi}{T} \times$ current sensitivity

The condition for the galvanometer to be ballistic: (1) the suspension wire should be fine or thin and the moment of inertia of the moving system should be large. (2) The coil should be wound on a non-conducting frame so as to provide small damping. (3) The circuit resistance and the resistance of the galvanometer must be large.

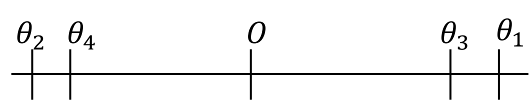
Concept of Dead beat galvanometer : When current is passed through a galvanometer, the coil oscillates about its mean position before comes to rest. To bring the coil to rest immediately, the coil is wound on a metallic frame. Now, when the coil oscillates, eddy currents are set up in a metallic frame, which opposes further oscillations of the coil. This inturn enables the coil to attain its equilibrium position almost instantaneously. Since the oscillations of the coil die out instantaneously, the galvanometer is called **dead beat galvanometer**.

The condition for the galvanometer to be dead beat : (1) the suspension wire should be thick and the moment of inertia of the moving system should be small. (2) The coil should be wound on a conducting frame so as to provide large damping due to eddy currents. (3) The circuit resistance and the resistance of the galvanometer must be small.

Damping : The process of decrease in amplitudes of successive oscillations of the suspension coil due to air resistance and induced emf developed in the coil is called damping.

Correction for damping in a BG

Logarithmic decrement : In a ballistic galvanometer, charge is measured by a sudden discharge due to which a sudden kick is given to the coil. Thus it is only the first throw that is effective in measuring the charge that flows through the coil. After the first throw, the coils oscillates with decreasing amplitudes.



Let θ be the actual throw in the absence of damping and $\theta_1, \theta_2, \theta_3, \theta_4, \theta_5 \ldots$ be the successive throws to the right and left continuously as shown. It is observed that $\frac{\theta_1}{\theta_2} = \frac{\theta_2}{\theta_3} = \frac{\theta_3}{\theta_4} = d$ where d is called the decrement and $log_e d$ is called the logarithmic decrement denoted by λ.

Thus $log_e d = \lambda \quad or \quad d = e^{\lambda}$.

Each complete oscillation comprises of two swings (i.e. from extreme right to left θ_1 *to* $\theta_2$and from extreme left to right θ_2 *to* θ_3).

$\frac{\theta_1}{\theta_3} = \frac{\theta_1}{\theta_2} \times \frac{\theta_2}{\theta_3} = d^2 = e^{2\lambda}$ (for two swings).

Similarly for four swings $\frac{\theta_1}{\theta_5} = d^4 = e^{4\lambda}$ and so on.

If θ be the actual throw in the absence of damping which is higher than the first observed throw θ_1. The motion of the coil from mean position to extreme right is half a throw. Thus $\frac{\theta}{\theta_1} = d^{\frac{1}{2}} = e^{\frac{\lambda}{2}} = \left(1 + \frac{\lambda}{2}\right)$ approximately.

Thus $\theta = \theta_1 \left(1 + \frac{\lambda}{2}\right)$.

Thus the expression for charge is $\boldsymbol{Q = \frac{T}{2\pi} \frac{c}{nAB} \theta_1 \left(1 + \frac{\lambda}{2}\right)}$

Calculation of $\boldsymbol{\lambda}$ The successive throws are noted and are related as

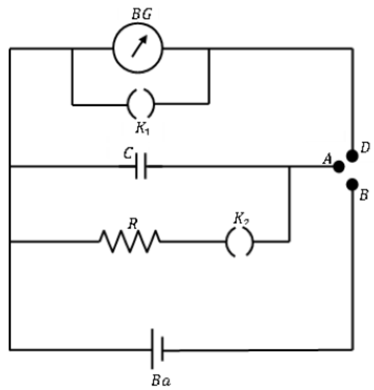
$\frac{\theta_1}{\theta_3} = e^{2\lambda}$, $\frac{\theta_1}{\theta_5} = e^{4\lambda}$, ... $\frac{\theta_1}{\theta_{11}} = e^{10\lambda}$, Thus taking logarithm on both the sides $log_e \left(\frac{\theta_1}{\theta_{11}}\right) = 10\lambda$ or $\lambda = \frac{2.303}{10} log_{10} \left(\frac{\theta_1}{\theta_{11}}\right)$.

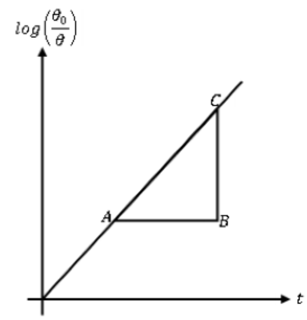
Thus by knowing the first and the eleventh throw, λ can be calculated. Thus

$$\boldsymbol{Q = \frac{T}{2\pi} \frac{c}{nAB} \theta_1 \left(1 + \frac{2.303}{20} log_{10} \left(\frac{\theta_1}{\theta_{11}}\right)\right)}$$

Determination of high resistance by leakage method

High resistance to be measured is connected in parallel with a capacitor and a ballistic galvanometer. The capacitor is charged to some suitable voltage (V_0) and is then allowed to discharge through the high resistance for a known interval of time (t). Remaining charge is discharged directly through the ballistic galvanometer. The deflection in the BG is proportional to the charge stored in the capacitor at that instant. Instantaneous voltage across the capacitor is given by $= V_0 e^{-\frac{t}{CR}}$. Using this, R can be calculated.





Circuit connections are made as shown in diagram. The lamp and scale arrangement is adjusted to get the reflected spot at the zero of the scale.

With key K_2 open and K_1 closed, A and B are connected and the capacitor is charged. The key between A and B is unplugged. Key K_1 is now opened. The capacitor is discharged through BG by connecting A and D. First throw θ_o is noted.

Capacitor is again charged as before. By closing the key K_2 , charge on the capacitor is allowed to leak through the high resistance for known interval of time (t) and remaining charge is discharged through the BG as before and the corresponding first throw θ_1 is noted.

From the theory of BG, $Q_0 = CV_0 = \frac{T}{2\pi}\frac{c}{nAB}\theta_0\left(1+\frac{\lambda}{2}\right)$

and $Q_1 = CV_1 = \frac{T}{2\pi}\frac{c}{nAB}\theta_1\left(1+\frac{\lambda}{2}\right)$ where V_0 and V_1 are the electric potentials across the capacitor. Therefore $\frac{Q_0}{Q_1} = \frac{V_0}{V_1} = \frac{\theta_0}{\theta_1}$. As $Q_1 = Q_0 e^{-\frac{t}{CR}}$

we have $\frac{Q_0}{Q_1} = e^{\frac{t}{CR}}$ or $log_e\left(\frac{Q_0}{Q_1}\right) = \frac{t}{CR}$ or $R = \frac{t}{C\, log_e\left(\frac{\theta_0}{\theta_1}\right)}$

or $\boldsymbol{R = \frac{t}{2.303C\, log_{10}\left(\frac{\theta_0}{\theta_1}\right)}}$. Knowing C, t, $\theta_0$and θ_1 R cam be calculated.

The experiment is repeated for different suitable times of leakage. In each case value of high resistance is found. A graph of $\log\left(\frac{\theta_o}{\theta}\right)$ against time of leakage (t) is plotted. Slope of the straight line is determined and hence the value of high resistance is calculated using formula$R = \frac{1}{2.303(slope)C}$.

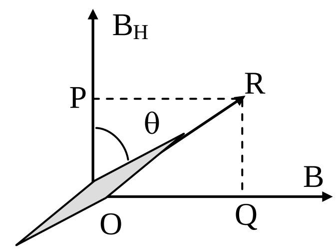
Apllications of BG

1. BG is used for the measurement of self inductance and mutual inductance of a coil or a pair of colis.
2. It is used to measure high resistances.
3. It is used ti compare caoacitances or to measure capacuatnce of a capacitor.

4. It is used in the determination of horizontal component of earth;s magnetic field.
5. It is used in the measurement of permeability and susceptibility of magnetic materials.

Magnetic meridian at a place is the vertical plane passing through the place and the magnetic north and south poles of the earth. The component of the earth's magnetic field along the horizontal in the magnetic meridian is called the Horizontal component of earth's field.

Tangent law: Consider a magnetic needle pivoted in a horizontal plane free to rotate in that plane. It comes to rest along the magnetic meridian. Its direction is along the horizontal component of earth's field B_H. Let another uniform magnetic field B is applied along the direction perpendicular to B_H. Now the magnetic needle deflects and comes to rest along the resultant of B and B_H, making an angle θ with the direction of B_H.



From the triangle OPR $\tan\theta = \frac{PR}{OP} \quad \therefore PR = OP\tan\theta \quad or \quad B = B_H \tan\theta$

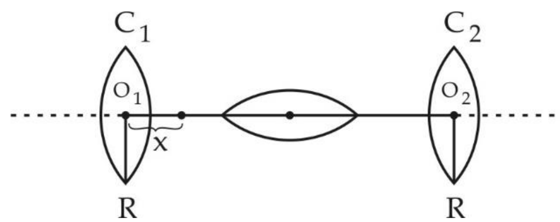
The above relation is known as the **tangent law**. It states that a magnetic needle pivoted in the region of two mutually perpendicular magnetic fields, alligns itself in the direction of the resultant of the two fields such that $B = B_H \tan\theta$ where θ is the angle between the direction of the resultant and B_H.

Helmholtz Tangent Galvanometer (HTG)

It consists of two identical circular coils mounted on the same base, placed coaxially at a separation equal to the radius of either coil. The two coils are connected in series and hence they carry same current in the same direction. A compass box is placed at the mid point between the coils along their common axis. As a point is moved away from this mid point, the increase in magnetic field due to one coil is compensated by the decrease in magnetic field due to the other and hence the field remains almost uniform over a small range around the mid point. Thus the compass needle deflects in a uniform magnetic field.

Theory of HTG

Let *I* be the current that flows through both the coils of HTG in same direction. Let the radius of each of them be r and number of turns be n. The distance between the coils is equal to radius of either of the coils. The magnetic field at a point at a distance x from one of the coils is $B = \frac{\mu_0}{4\pi}\frac{2\pi n I r^2}{(r^2+x^2)^{3/2}}$



or $\quad B = \dfrac{\mu_0 n I r^2}{2(r^2+x^2)^{3/2}}$

If the magnetic field in a region along the common axis is uniform then,

$\frac{dB}{dx} = constant \therefore \frac{d^2B}{dx^2} = 0$

$$\frac{dB}{dx} = \frac{d}{dx}\left[\frac{\mu_o}{4\pi}\frac{2\pi n I r^2}{\left(r^2+x^2\right)^{3/2}}\right] = \text{constant}$$

$$\Rightarrow \quad \frac{\mu_o n I r^2}{2}\left(-\frac{3}{2}\right)\left(r^2+x^2\right)^{-5/2}(2x) = \text{constant}$$

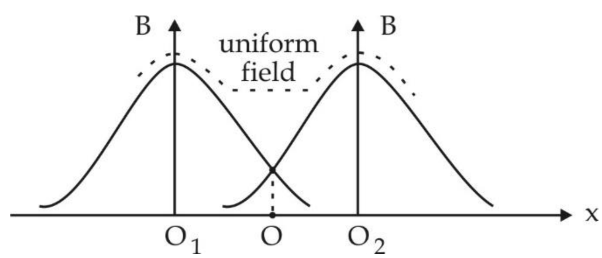
or $-\frac{3}{2}\mu_o n I r^2 x\left(r^2+x^2\right)^{-5/2} = \text{constant}$

$$\frac{d^2B}{dx^2} = -\frac{3}{2}\mu_o n I r^2\left[x\left(-\frac{5}{2}\right)\left(r^2+x^2\right)^{-7/2}(2x) + \left(r^2+x^2\right)^{-5/2}\right] \Rightarrow \quad \left(r^2+x^2\right)^{-7/2}\left[-5x^2+r^2+x^2\right] = 0$$

$\Rightarrow \quad r^2 = 4x^2 \quad \Rightarrow \quad r = 2x \quad \Rightarrow \quad \boxed{x = \frac{r}{2}}$ i.e., the magnetic field is uniform around the mid point between the two coils. We have $B = \frac{\mu_o n I r^2}{2\left(r^2+x^2\right)^{3/2}}$ As there are two coils, this equation must be multiplied by 2. Also substituting for x as $x = \frac{r}{2}$, we have

$$B = \frac{2\mu_o n I r^2}{2\left(r^2+\frac{r^2}{4}\right)^{3/2}} = \frac{\mu_o n I r^2}{\left(\frac{5r^2}{4}\right)^{3/2}}$$

$$= \frac{\mu_o n I r^2}{5\sqrt{5}\frac{r^3}{8}} = \frac{8}{5\sqrt{5}}\frac{\mu_o n I}{r}$$



Let B_H be the horizontal component of earth's magnetic field and let the plane of the coils be in magnetic meridian so that B and B_H are perpendicular to each other. Let θ be the deflection of the magnetic needle with respect to B_H.

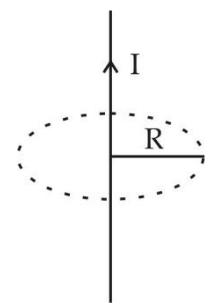
Then, from tangent law $B = B_H \tan\theta$. Substituting for B in the equation $B = \frac{8}{5\sqrt{5}} \frac{\mu_0 n I}{r}$

$\Rightarrow \quad \frac{8}{5\sqrt{5}} \frac{\mu_o n I}{r} = B_H \tan\theta \Rightarrow \quad \boxed{I = \frac{5\sqrt{5}}{8} \frac{r B_H}{\mu_o n} \tan\theta} \Rightarrow \quad I = K \tan\theta$, where K is called the Reduction factor of HTG given by $k = \frac{5\sqrt{5}}{8} \frac{r B_H}{\mu_0 n}$

Thus ***I ∝ tan θ.***

Ampere's circuital law

Statement The line integral of the magnetic induction $\vec{B}$ around any closed path is equal to μ_o times the net current across the area bounded by the path.



Consider an infinitely long conductor carrying current I. The lines of magnetic induction around the conductor are circular and are in a plane perpendicular to the plane containing the conductor.

The magnetic induction at a distance R from the conductor is given by $B = \frac{\mu_o I}{2\pi R}$

The line integral along the closed path of radius R is $\oint \vec{B} \cdot \vec{dl} = \int B\, dl = \int \frac{\mu_o I}{2\pi R} dl$.

{∵ angle between B and dl is zero}.

$$\Rightarrow \quad \oint \vec{B} \cdot \vec{dl} = \frac{\mu_o I}{2\pi R} \int dl$$
$$= \frac{\mu_o I}{2\pi R} \times 2\pi R = \mu_o I$$

$\Rightarrow \quad \boxed{\oint \vec{B} \cdot \vec{dl} = \mu_o I}$ Hence the statement.

This statement holds good even when the closed path is not circular.

Applications

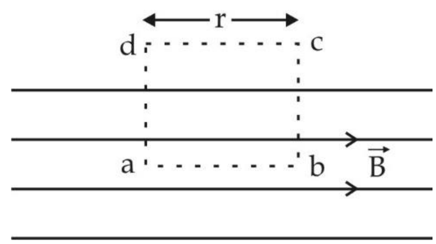
I. Field at a point due to a straight current carrying conductor:

Consider a straight conductor carrying current I. It is desired to find magnetic field B at a point distance R from the conductor. From Ampere's circuital law $\oint \vec{B} \cdot \vec{dl} = \mu_o I \Rightarrow \int B\, dl = \mu_o I$ (since $B \cdot dL = Bdl\, cos\theta = B\ dl\ as\ \theta = 0$)

$\Rightarrow B\int dl = \mu_o I \quad \Rightarrow \quad B \times 2\pi R = \mu_o I$ (as for a closed path $\int dl = 2\pi R$)

$\therefore \boxed{B = \dfrac{\mu_o I}{2\pi R}}$

II. Field at a point on the axis of a solenoid:



Consider a uniformly wound solenoid carrying a current I. The magnetic field inside the solenoid is parallel to its axis and it is the vector sum of the fields set up by all the turns of the solenoid.

Consider a closed path $abcd$ as shown in the figure.

Applying Ampere's law to this path,

$\oint \vec{B} \cdot \vec{dl} = \mu_o I'$, where I' is the net current across the closed path.

This integral can be written as

$$\oint \vec{B} \cdot \vec{dl} = \int_a^b \vec{B} \cdot \vec{dl} + \int_b^c \vec{B} \cdot \vec{dl} + \int_c^d \vec{B} \cdot \vec{dl} + \int_d^a \vec{B} \cdot \vec{dl}$$

Along bc and da, $\vec{B}$ and $\vec{dl}$ are perpendicular to each other and hence,

$\int_c^d \vec{B} \cdot \vec{dl} = 0$ and $\int_d^a \vec{B} \cdot \vec{dl} = 0$

cd is outside the solenoid and hence it is ineffective ie., $\vec{B}$ along cd due to the current in the solenoid is zero.

$\therefore \int_c^d \vec{B} \cdot \vec{dl} = 0$ $\therefore$ The total field inside the solenoid is only due to the path ab.

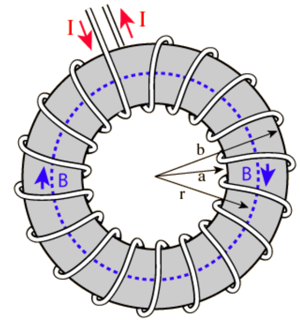
$$\therefore \oint \vec{B} \cdot \vec{dl} = \int_a^b \vec{B} \cdot \vec{dl} = \int_a^b B\, dl = B\int_a^b dl \quad \oint \vec{B} \cdot \vec{dl} = Br \{\because ab = r\}$$

If n is the number of turns per unit length of the solenoid, the total number of turns in a length r is nr.

$\therefore$ Net current bounded by the path $abcd$, $I' = nrI$.

Applying Ampere's law, $\oint \vec{B} \cdot \vec{dl} = \mu_o I' \Rightarrow Br = \mu_o nrI$

$\therefore \boxed{B = \mu_o nI}$



III Magnetic field due to a toroid A toroid is a coil of insulated or enameled wire wound on a donut-shaped form made of powdered iron. A toroid is used as an inductor in electronic circuits, especially at low frequencies where comparatively large inductances are necessary.

Consider a toroid of mean radius r and n be the number of turns per unit length. Consider the closed path in the form of circle enclosing the turns (dotted circle). Along this circle the value of B is uniform and its direction is tangential to the circle.

The line integral of B over the closed path is $\int B.dl = B\int dl = B.2\pi r$ ....(1)

From Ampere's circuital law

$$\int B.dl = \mu_0 \times current\ enclosed\ by\ the\ closed\ path$$

$\int B.dl = \mu_0 \times n\,2\pi r\,I$ ......(2)

Comparing equations (1) and (2), we get $B.2\pi r = \mu_0 \times n\,2\pi r\,I$

or $\boldsymbol{B = \mu_0\, n\, I}$

If N is the number of turns in the toroid, then the equation is $B = \frac{\mu_0 N\,I}{2\pi r}$.

PART-A

1. (a) Explain magnetic field in terms of magnetic lines of force.
   (b) Write the general expression for the force acting on a moving charge in a magnetic field Discuss conditions for the force to be maximum and minimum?
   (b)Define magnetic field based on the force on a charged particle and define it's SI unit
2. (a) Obtain an expression for the force on a current carrying conductor placed in a uniform magnetic field. Give the condition for maximum and minimum force
   (b) State Ampere's swimming rule and Right hand clasp rule
3. (a) Derive an expression for the force between two parallel and infinitely long conductors carrying current and hence define ampere
   (b) Explain Fleming's left hand rule.
4. (a)State and explain Biot – Savart's law and hence arrive at the law.
5. (a) Derive an expression for the magnetic field at a point due to straight conductor of finite length carrying current. What will be the expression for magnetic field if it's length is infinite
6. Derive an expression for the magnetic field at a point along the axis of a circular coil carrying current. Show the variation graphically. What is the expression for the field at the centre of the coil?
7. (a) Show that net force acting on a coil carrying current placed in a magnetic field is zero.

(b) Derive an expression for the torque acting on a rectangular current loop of **N** turns, with the normal of the loop at an angle θwith respect to the magnetic field **B.**

(c) when is the torque maximum and minimum?

8 Derive an expression for the magnetic field at a point along the axis of the solenoid carrying current. Hence deduce the expressions for the magnetic fields at the centre and one of the ends along the axis of infinitely long solenoid.

9 Give the principle and theory of a Ballistic Galvanometer. Define ballistic reduction factor or charge sensitivity

10 (a) Give the conditions under which the ballistic galvanometer is dead beat. Define logarithmic decrement. What is the correction for damping?

(b)With necessary theory, describe an experiment to determine the high resistance by Leakage using a ballistic galvanometer

11 Give the theory of Helmholtz double coil galvanometer and hence define reduction factor of the galvanometer. Also represent graphically the variation of the field along the axis of the double coil.

12 State and explain Ampere's circuital law. (a) Using Ampere's circuital law, deduce an expression for magnetic field inside a toroid carrying current.

(b) Using Ampere's circuital law, deduce an expression for magnetic field along the axis of the solenoid carrying current.

(c) Using Ampere's circuital law, deduce an expression for magnetic field at a point due to a straight conductor carrying current.

PART-B

1. A uniform magnetic field of 1.7 T points horizontally from south to north. A proton of energy 3.5 MeV moves vertically downwards through this field. Calculate the force on the proton. (Mass of proton=$1.67 \times 10^{-27} kg$ and $e = 1.6 \times 10^{-19} C$.

   [Hint : $F = e\,v\,B\,sin90$, and $E = \frac{1}{2}mv^2$, $find\ v. Also\ E = 3.5MeV = 3.5 \times 10^6 \times 1.6 \times 10^{-19} J$ $F = 6.98 \times 10^{-12} N$]
2. A proton is moving northwards with a velocity $5x10^6 ms^{-1}$ in a magnetic field of 0.1T directed eastwards. Find the force on the proton. Given charge on proton=1.6×10^{-19}

   [Hint : $F = e\,v\,B\,sin90 = 0.8 \times 10^{-13} N$]
3. A long straight wire carries a current of 20 A. An electron travelling at 10^7 ms^{-1} speed is 0.02 m from the wire. What force acts on the electron if its motion is towards the wire

   [Hint : $B = \frac{\mu_o I}{2\pi R} = 2 \times 10^{-4} T$, $Direction\ of\ B\ is\ right\ angles\ to\ electron\ motion,$ $F = e\,v\,B\,sin90 = 3.2 \times 10^{-16} N$]
4. Calculate the force on a wire of length 0.1m in which a current of 8A is flowing ,if the wire is kept at an angle of (i) 30° , (ii) $90^0$and (iii) 0° to the direction of field of 1T.

   [Hint : $F = BIL\,sin\theta$, $(i)\ 0.4\ N$, $(ii)\ 0.8\ N$ (iii) zero]
5. Two long wires P and Q are held perpendicular to the plane of the paper with a distance of 5m between them. If P and Q carry currents of 2.5 A and 5A respectively in the same direction calculate the magnetic field at a point half-way between the

wires.

[Hint : $B_1 = \frac{\mu_o I_1}{2\pi R}$ and $B_2 = \frac{\mu_o I_2}{2\pi R}$ where $R = 2.5\ m,\ B = B_2 - B_1 = 2 \times 10^{-7}T$

6. The two long straight wires in each carry a current of I = 5 A in opposite directions and are separated by a distance d = 30 cm. Find the magnetic field a distance l = 20 cm to the right of the wire on the right.

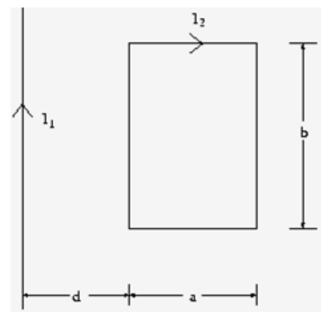
[Hint : $B_1 = \frac{\mu_o I_1}{2\pi R_1}$ where $R_1 = 30 + 20\ cm$ and $B_2 = \frac{\mu_o I_2}{2\pi R_2}$ where $R_2 = 20\ cm,\ B = B_2 - B_1 = 30 \times 10^{-7}T$

7. Two infinitely long straight conductors carrying a current of 10A each are separated by a distance of 0.01m. Find the force acting per each metre of conductor by the other.

[Hint : $F = \frac{\mu_0 I_1 I_2}{2\pi r}$ $F = 2 \times 10^{-3}\ N$]

8. Find the force of attraction on a straight conductor of length 10cm carrying a current of 5A kept parallel to another straight long conductor carrying a current of 8A in the same direction at a distance of 2cm.

[Hint : $F = \frac{\mu_0 I_1 I_2}{2\pi r}$ $F = 40 \times 10^{-4}\ N$]



9. For the arrangement shown in Fig the long straight wire carries a current of I_1 = 5 A. This wire is a distance d = 0.1 m away from a rectangular loop of dimensions a = 0.3 m and b = 0.4 m which carries a current I_2 = 10 A. Find the net force exerted on the rectangular loop by the long straight wire.

[Hint : $F_{12} = \frac{\mu_0 I_1 I_2 L}{2\pi r_1}$, where $r_1 = 0.1m$, $F_{13} = \frac{\mu_0 I_1 I_2 L}{2\pi r_2}$ where $r_2 = 0.3 + 0.1m$ and $l = b = 0.4m$, $F_R = F_{12} - F_{13} = 3 \times 10^{-5}N$]

10. Calculate the strength of the magnetic field at the centre of the coil of radius 0.1m and having 50 turns carrying a current of 1A.

[Hint : $B = \frac{\mu_0}{4\pi}\frac{2\pi nI}{r}$ $B = 3.14 \times 10^{-4}T$]

11. A coil of radius 0.1m and 100 turns carries a current of 2A. Find the magnetic field at a point 5cm from the centre of the coil on its axis

[Hint : $B = \frac{\mu_0}{4\pi}\frac{2\pi nIr^2}{(r^2+x^2)^{\frac{3}{2}}}$ $B = 1.93 \times 10^{-3}\ T$]

12. Two circular coils of radius 0.1meach having 20 turns are mounted co-axially at a distance of 0.1m apart. A current of 0.5 A is passed through both of them (i) in the same direction and (ii) in the opposite direction. Find the magnetic induction at the centre of each coil.

[Hint : $B = \frac{\mu_0}{4\pi}\frac{2\pi nIr^2}{(r^2+x^2)^{\frac{3}{2}}}$ when currents are in the same direction, $B = B_1 + B_2 =$

$8.5 \times 10^{-5}\,T$, when the currents are in opposite direction, $B = B_1 - B_2 = 4.06 \times 10^{-5}\,T$]

13. A solenoid of length 4 m and radius 0.02 m wound uniformly with 1000 turns of wire. If it carries a current of 2 A, what is the value of magnetic field (i) on the axis of solenoid at the centre (ii) on the axis at one end.

    [Hint : at the centre $B = \frac{\mu_0 NI}{l} = 3.14 \times 10^{-4} T$ *and at one end along axis*, $B = \frac{\mu_0 NI}{2l} = 1.57 \times 10^{-4} T$]

14. A coil of area 50cm^2 having 300 turns carries a current of 5mA,is suspended in a magnetic field of strength 2x10^{-2}T. Find the torque acting on the coil if the plane of the coil makes an angle 60° with the direction of magnetic field.

    [Hint : $\tau = M\,B\,\sin\alpha$, $M = N\,I\,A$, α = 90-60 = 30, $\tau = 7.5 \times 10^{-3}$ *Mm*]

15. A capacitor of capacitance 500pF is charged to a potential of 1.5V and then discharged through a ballistic galvanometer. The first throw noted on a scale placed away is 0.6m. If the time period of oscillation is 5s and the logarithmic decrement is 0.02, calculate the ballistic constant and the figure of merit of the galvanometer.

    [Hint : $Q = CV$, $\frac{c}{nAB} = k$ called figure of merit,

    $Q = \frac{T}{2\pi}\frac{c}{nAB}\theta_1\left(1 + \frac{\lambda}{2}\right)$. $Q = K\theta_1\left(1 + \frac{\lambda}{2}\right)$ or $K = \frac{2\pi}{T}\frac{Q}{\theta_1\left(1+\frac{\lambda}{2}\right)} = 1.554 \times 10^{-9}$. Figure of merit $= \frac{q}{\theta} = k = \frac{T}{2\pi} \times K = 1.237 \times 10^{-9} Cm^{-1}$]

16. The successive throws on the same side of the mean position for an oscillating coil is 25,24.9 and 24.8 cm. Calculate the logarithmic decrement.

    [Hint : For accurate values first and last throws are taken. $\frac{\theta_1}{\theta_5} = d^4 = e^{4\lambda}$ *Thus* $4\lambda = log_e\left(\frac{\theta_1}{\theta_5}\right)$ *or* $\lambda = \frac{2.303}{4} log_{10}\left(\frac{25}{24.8}\right) = 0.002$

17. A Helmholtz galvanometer has coils of radius 0.11 m and number of turns 70 √5. Calculate the current through the coils which produces a deflection of 45°. What will be the deflection if the current is doubled? B_H = 0.32 × 10^{-4} T

    [Hint : ⇒ $\boxed{I = \frac{5\sqrt{5}}{8}\frac{r B_H}{\mu_o n}\tan\theta}$ $I = 2.5 \times 10^{-2} A$ $I_1 = K\,tan\theta_1$ *and* $I_2 = K\,tan\theta_2$

    $I_2 = 2I_1$ $K\,tan\theta_2 = 2\,K\,tan\theta_1$ Thus $tan\theta_2 = 2\,tan\theta_1$, θ_2 = 63.43^0]

18. An electron accelerated under a potential difference of 5 kV enters a magnetic field of 2T. Calculate the radius of the circle described if the field is transverse to the direction of velocity of the electron. $m - 9.1 \times 10^{-31}\,kg$

    [Hint : $\frac{1}{2}mv^2 = eV$, *Find v*, $F = evB = \frac{mv^2}{r}$, *r is gevn by* $r = 1.19 \times 10^{-4} m$]

19 A beam of electros travels through a region where the magnetic field is 50mT and electric field is 25kVm^{-1}. If the path of the beam is not changed, find the speed of electron beam. If the electric field is removed, what is the radius of electron beam described?

[Hint : $qvB = qE$, *find v* $R = \frac{mv}{eB} = 5.68 \times 10^{-5} m$]

20 Two identical circular coils of radii 0.1m having 100 turns each placed such that they are concentric and the planes are perpendicular to each other. If the current

is 2A in each coil, find the resultant magnetic field at their common centre. What is the direction of the magnetic field?

[Hint : $B_R = \sqrt{B_1^2 + B_2^2}$ where $B_1 = B_2 = \frac{\mu_0}{4\pi}\frac{2\pi nI}{r}$ $B_R - 1.77 \times 10^{-3}T$

21 A solenoid of length 1.5 m has a radius of 2 cm which contains three windings each of 1200 turns, If the current in the solenoid is 8A calculate the magnetic field at its centre. What is the magnetic flux through the cross section at the same point?

[Hint : $B = \frac{\mu_0 N\, I}{l} = 3.62 \times 10^{-2}T.$ $Magnetic\ flux\ \phi = BA = B\pi r^2 = 4.55 \times 10^{-5} wb$]

22 A capacitor of $0.5\mu F$ *is charged to* $4V$ and discharged through BG which produces a deflection of 20 cm.Calculate the current sensitivity of the galvanometer. Given time period of BG is 15s.

[Hint : $Q = CV,\ Current\ sensitivity = \frac{T}{2\cdot\pi} \times charge\ sensitivity = \frac{T}{2\cdot\pi} \times \frac{\theta}{Q} = 2.38 \times 10^5\ mA^{-1}$]